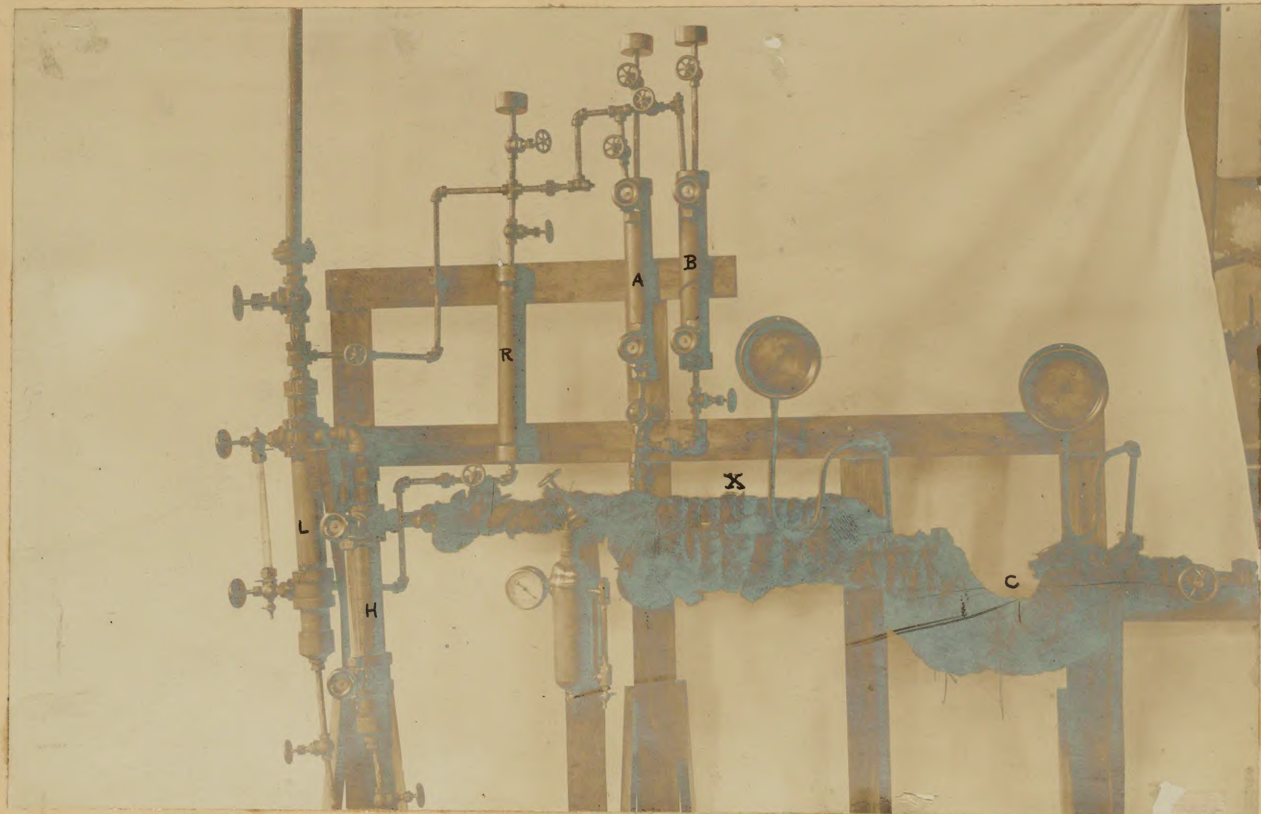
25

The Design and Construction
of an Apparatus for Proving
Napier's Formula.

Morris Peterzell

June 11/1902

20 May '48. Sec. G. Univ. P. M.



1.

The formula $W = \frac{PA}{70}$, popularly known as "Napier's Formula", dates as far back as 1867 when Mr. R.D. Napier, one of the able correspondents of the "London Engineer", announced the formula for the flow of steam:- $W = kA\sqrt{\frac{P_0(P-P_0)}{PB}}$
where W is the weight per minute; k is a constant; P, the external higher pressure in pounds per square inch; P_0, the lower pressure; B, the specific volume of the steam before it reaches the orifice; and A is the area of the orifice in square inches.
The experimental value of k, he found for thin plates to be about 275. This formula, he said, will hold true if P_0 is about one half P or below this value. But if it is above $\frac{1}{2}P$, the following formula will hold:- $W = \frac{k}{2}A\sqrt{\frac{P}{B}}$.

Mr. Napier was immediately rigorously attacked by various writers and experimenters upon

the subject, showing how ridiculous a formula of this character was; that in some cases it gives results which do not agree with either theory or experiment. Napier produced defences that his formula is not strictly mathematically correct, but is true for all practical purposes; that he derived the formula mainly from reasoning but confirmed by subsequent experiments. In some of his defences against the attacks upon him, he gradually explains how he came to the conclusions. Some of his explanations or reasons for assuming the formula, quoting them in his own words, are:-

" If steam is being discharged from a boiler through a tube, all that escapes through the outer end must have entered the other end of it, but the only cause competent to make the steam rush from the boiler into the tube is a difference of pressure in the tube and boiler; therefore, other things being equal, the velocity from the boiler must depend on the difference of pressure in the tube and that in

the boiler."

"If fluids of different densities, but otherwise alike, are discharged under similar circumstances, then, if the discharging pressures are proportional to the densities, the velocities will be the same."
"From the first proposition it is clear that so long as the density and pressure in both, the boiler and tube, remain constant, there can be no alteration in the quantity of steam passing through the tube".

In another article, he describes some of his experiments as follows:—"My experiments were of two kinds: the first consisted in ascertaining the weight discharged, and the second in ascertaining the elastic force in the different parts of the experimental tube. These were quite sufficient of themselves to have established the law of the velocity of the steam being acquired when the outer pressure is half that of the inner or boiler pressure. The weight discharged was calculated from the amount of heat imparted to water contained in

an iron bucket, whose water equivalent was first ascertained. The plan adopted was as follows:— the outer end of a tube, about $1\frac{3}{4}''$ in diameter, leading from the boiler, was stopped up with a plate; into this plate, the experimental tube, generally about $\frac{1}{2}''$ diameter was screwed; the outer end of the experimental tube was screwed into the end of another tube, much larger than the experimental tube. This last outer tube was led by a bend into the bucket of water, and the temperature of the water having been taken, the steam was turned on, and P being the pressure in the boiler, P_0 was that taken in the outer tube, a short distance before the end of the experimental tube (I had also an arrangement for preventing spluttering or noise during the experiment). Exactly at the end of a minute, the cock was shut and the temperature then taken by a thermometer that had remained in the water during the experiment. By partially stopping up the outer end of the outer or conducting

pipe, the pressure P_o would be increased in it; by leaving it open, there would be a partial vacuum in it; and by this means I saved the outer pressure."

He then gives experimental results and data and compares them with those calculated from his formula; and these agree very closely.

But all these explanations and experiments of Napier, even his subsequent modification of his first formula which he extended to orifices in plates instead of short tubes, did not calm Mr. J. Baldwin, the "Doctor of Gases", in those days. But there came a man, W.J.M. Rankine, whose reputation as a scientist and discoverer no one contradicted, even Mr. Baldwin. He began to investigate Napier's formulae and experiments, and comparing them with his own theoretical, complicated thermodynamic formulae, and also those of Weisbach's, he published in the same periodical the results of his researches upon Napier's formulae. He averages up all of Napier's results with those of his own and those

of Zeuner and from these he makes the following conclusions:- "When dry saturated steam is supplied during its expansion with heat just sufficient to prevent liquefaction, the maximum discharge per second expressed in pounds is obtained by taking $\frac{1}{70}$ part of the absolute internal pressure on a given area".

"Conclusions: From the general agreement of the results of Mr. R.D. Napier's formulae with those of his experiments, and of both with those of theoretical formulae, taken in combination with the supposition that the pressure at the throat of the outlet never falls below that corresponding to the maximum mass-velocity, it may be inferred that the following conclusions, if not absolutely proved, are at all events highly probable".

"First: the pressure at the throat of the outlet never falls below that corresponding to the maximum mass-velocity of outflow, how low soever the external pressure may be; and, so far as I know, the merit of originally proposing and applying this principle

belongs to Mr. R. D. Napier."

"Secondly: a rule based on the combination of the preceding principle with thermodynamic formulae and tables for the work of expanding steam, gives results, nearly agreeing with those of Mr. Napier's experiments and of his second formula."

"Thirdly: Mr. Napier's pair of formulae, give results which are good approximations for practical purposes."

"Fourthly: Outflow in units of weight per unit area of throat, nearly:

When $p_1 \geq \frac{5}{3} p_2$

$$\text{Outflow} = p_1 \div 70$$

and when $p_1 < \frac{5}{3} p_2$

$$\text{Outflow} = \frac{p_2}{42}\sqrt{\frac{3(p_1 - p_2)}{2 p_2}}$$

W. J. M. R.

Nov 3/1869.

The object in designing this apparatus was to prove experimentally Napier's formula; i.e. to determine the constant in:—

$$W = \frac{PA}{K}$$

for dry saturated steam under various external and internal pressures; to find a law for the discharge of steam through orifices, when containing different amounts of moisture; and also, the discharge when the steam is superheated: The apparatus was, therefore, designed to fullfil all these conditions.

We will first begin with a general description of the apparatus, and then a detailed description of the different parts and their object, and how to manipulate them when performing the various researches.

From the picture on the first page will be seen, a vertical pipe on the left, which supplies the steam for the apparatus. At the end of this pipe there is a globe valve which may be used for two purposes:— to admit or shut off the steam or to throttle it for the purpose of reducing the upper pressure. The steam now enters L. The object of the latter is to convert the steam, which becomes superheated on being throttled when passing through the above mentioned valve, into a saturated condition, by bringing it in contact with ~~cool~~ water, which is supplied to it from R. The steam now enters H which is a separator, depositing there the excessive moisture it carries, after which it proceeds through a horizontal pipe, to three horizontal pipes connected to each other by return bends. It now enters C which contains a brass plate with an orifice at its center.

Adjacent to this plate, and on either side of it, is an oil-well for a thermometer. On either side of the oil-well is a goose-neck bearing a pressure-gauge. On the extreme right is a globe valve used for varying the lower pressure. After passing this valve, the steam is led by a pipe into a surface condenser where it is condensed and flows out through a vertical pipe in the bottom of the condenser.

<u>L</u> consists of a 2" pipe with crosses; in the outer ends of the latter are nipples of the same diameter as the pipe; and on the upper extremity of the upper nipple is a reducer from 1" to 2"; from the lower nipple there issues a $\frac{1}{4}$" drain pipe, with a valve, which is connected to the above through a reducer. Into one of the arms of each of the crosses are screwed in gauge glass fittings, the distance between the centers of which is 12". The pipe which is screwed in the upper reducer extends downward into the

body of L a distance of 2" below the lower edge of the upper cross, so that the steam issuing from the 1" pipe into L is compelled to touch the water therein which should be kept several inches below the end. When the superheated steam comes in contact with the water, the latter is evaporated and carries with it some water which is deposited in H which is made similar to L. The water is supplied to L from R which is a 2" pipe 15" long with caps at each end. The upper cap has three holes drilled and tapped in it; one for communication with the steam from the supply pipe through quarter inch pipes which are connected to the former by means of a reducing tee on it; into another hole is screwed in a quarter inch nipple which bears a valve and on the top it has a 2" cap screwed on to serve as a funnel to fill R with water; into the third hole is screwed in a cock for allowing the air to escape when R is being filled. Into the lower cap is screwed in

a quarter inch pipe which allows the water from R to flow into L, when the valve on it is opened. On account of the connection of R with the steam supply pipe, the flow of water from it into L is due to the head of water only, and by opening communication between the two, when the water in L is low, as indicated by the gauge-glass in the latter, water is allowed to flow into it until the level reaches about two or three inches below the end of the pipe inside it, when the valve should be closed.

About fifteen inches to the right of the separator is a reducing tee into which is screwed in a ½" nipple at right angles to the direction of the main steam flow and in a horizontal plane with it. This nipple is attached through a valve to a carpenter Calorimeter. The latter is for determining the quality of the steam used.

About three inches to the right of the tee for the Calorimeter, is another tee into which

is screwed a $\frac{1}{4}''$ nipple in a vertical plane with the steam line, and this divides up by means of a tee into two branches, which contain valves, and lead to two water reservoirs A and B. These are like the water receptacle R but have for their object to supply water to the steam, in order to produce steam of different qualities, according to the amount of water let in. These reservoirs, as can be seen from the picture are similar in construction to that of R, and by means of an extension of the $\frac{1}{4}''$ pipes which communicate with the steam in the vertical pipe, are supplied with steam from the same source, above the water to balance the steam pressure below the water, so that the flow is determined by the height of the water in the reservoirs. It will be seen that there is a valve between A and B, in the horizontal pipe above them, and one in the vertical pipe of A which joins the horizontal one by means of a $\frac{1}{4}''$ tee.

The object of having two reservoirs is to have the flow of water from them, continuous; therefore, when one is empty, the other is allowed to run while the first is filled again. To do this, the steam above the water has to be shut off from this reservoir, and the same is true when the other has to be filled; therefore, by properly manipulating these valves, this result can be accomplished.

In order to determine the amount of water that gets in the steam, the reservoirs require to be calibrated. This can be done by marking the gauge-glass, in each, at the lowest point, then pouring in a known weight of water, through the funnels above them, until the highest convenient point of the glass. Then, since the reservoirs are of uniform cross-section, the distance between the two marks can be divided up conveniently into equal portions on a scale fixed alongside of the glass, and a pointer which

shall slide on the glass, will indicate correctly the height of the water in the glass, which is the same as in the reservoir.

When pouring water into the reservoirs, the air cocks in the upper caps, should be opened in order to leave the air out. There is also an arrangement of valves in the two branches leading from the steam line into the reservoirs, for the purpose of shutting off from the steam, either both reservoirs or else only one.

In order, therefore, to prove or to see the different effects moisture in the steam has upon the discharge through an orifice, we determine the quality of the steam entering the apparatus by means of the calorimeter, and to the moisture thus determined, we add the weight of the water admitted, in a certain time, from the reservoirs.

It was experimentally shown by professor Jacobus in one of his papers, for the

A.S.M.E., that in a horizontal pipe carrying steam, the more moist steam, or the water in the steam, is carried along the bottom of the pipe. Also, in that same paper, he describes a method for obtaining moist steam, by means of a water-jacket around a portion of the steam pipe. The former reason has been adhered to, by attaching the calorimeter in such a manner, so that it should not take the steam from the top of the pipe which is dryer, nor from the bottom of the pipe which is wetter than the average steam in the pipe. Therefore, the calorimeter has been so connected to the steam line, so that it would take steam from the center or middle of the pipe, which would probably represent the average condition of the steam. Also, with reference to imparting moisture to the steam by Jacobus' method, involves not only the weighing of the water entering or issuing from the jacket,

which involves some inaccuracies, for some water might leak out on the way through the jacket, but also the determination of the temperature of the water prior and subsequent to entering the jacket. The scheme used in this apparatus, obviates all the inconveniences mentioned.

It was mentioned in the brief description of this apparatus, that the steam is compelled to pass through three horizontal pipes connected by return bends, before it reaches the first pressure gauge. These pipes, marked X, are arranged so that the plane of the three is vertical, and as the water from the reservoirs enters the steam, it is hurled in a zigzag motion, so as to mix up thoroughly the water with the steam. These pipes can also serve another purpose; viz, for experimenting upon the discharge of perfectly dry or superheated steam through an orifice. In that case they can be used as a superheater by placing a large bunsen burner beneath them.

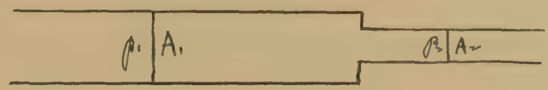
At the extremity of the third or bottom pipe is a tee for the goose-neck of the first pressure gauge. At the right end of this tee is a nipple connecting the former with a reducing tee containing an oil-well for a thermometer. This latter portion of the apparatus is illustrated on the next page. It will be seen that the tee increases for a short length the sectional area of the steam passage. This sectional increase is necessary to compensate for the diminution of the otherwise resulting area due to the oil-well. The steam impinging or coming in contact with the brass tube containing heavy lubricating oil, heats it, the temperature being recorded by the thermometer held in the oil. Now, were we not using the sizes of nipples and fittings shown, but had used inch pipes for this portion, the sectional area of the steam passage would be reduced to half or more by the oil-well and the resulting steam coming

suddenly into a narrow passage is throttled and becomes superheated, and, therefore, the temperature recorded by the thermometer would be higher than was before. After passing the oil-well (first, to the left), the steam enters again an inch nipple from where it passes through the orifice in a brass plate placed between the flanged union. The plate and bolts in the union are insulated with asbestos, and the whole length of pipe work, beginning with the rejion near the calorimeter up to the second pressure gauge, is wrapped in hair-felt to prevent as far as possible radiation. The use of the thermometers in conjunction with the gauges is to determine the theoretical discharge and quality of steam.

In order to show clearly how this is effected, we will derive and consider a thermodynamic formula for the discharge of steam through an orifice.

Imagine in any pipe carrying steam

two pistons A_1, A_2.



The energy at one portion must equal to the energy in the second.

Assuming general conditions, the energy at A_1 is

$$p_1V_1 + H_1 + \frac{\omega_1^2}{2g} + R$$

where p_1V_1 is the work done upon the gas by the gas on the left of an imaginary piston A_1.
H_1, is the heat contained in the gas, or its intrinsic energy; $\frac{\omega_1^2}{2g}$ is the kinetic energy of the gas in this section of the pipe or tube; and R is the heat received or given out by the gas, externally.

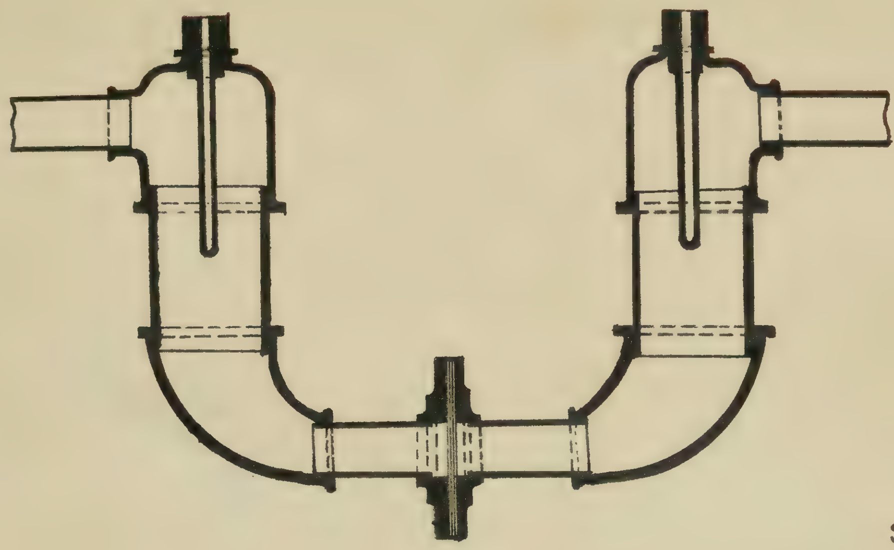
The energy at A_2 is likewise:—

$$p_2V_2 + H_2 + \frac{\omega_2^2}{2g}$$

$$\therefore\ p_1V_1 + H_1 + \frac{\omega_1^2}{2g} + R = p_2V_2 + H_2 + \frac{\omega_2^2}{2g}$$

which is the general equation for the flow of perfect gases.

For the case of steam, the same reasoning as above might be applied and the equation assumes



Sectional View
of Oil-Wells and Plate.
Scale-$\frac{1}{4}''=1''$

the form:—

$$A p_1 u_1 + (q_1 + x_1 \rho_1) + \frac{\omega_1^2}{2g} + R = A p_2 u_2 + (q_2 + x_2 \rho_2) + \frac{\omega_2^2}{2g}$$

where $A p_1 u_1$ is the work done upon or by the steam, A is the reciprocal of Joules equivalent; $q_1 + x_1 \rho_1$, is the heat contained in the steam; x_1, the portion of steam;

A similar interpretation is also applicable to the terms of the second member of the equation.

By transposition and combination, we get, neglecting the small quantity $\frac{\omega_1^2}{2g}$

$$\omega_2^2 = 2g\left[(q_1 + x_1 r_1) - (q_2 + x_2 r_2) + R\right]$$

If no heat is received or given out; i.e. if the steam expands adiabatically, $R = 0$

and $\therefore \omega_2^2 = \left[(q_1 + x_1 r_1) - (q_2 + x_2 r_2)\right] 2g$

If a denotes the area of the orifice in square feet, then the quantity in cubic feet discharged per second is,

$$Q = a\sqrt{2g\left[(q_1 + x_1 r_1) - (q_2 + x_2 r_2)\right]}$$

In order, therefore, to have adiabatic expansion, we must prevent radiation as far as possible. Suppose the first gauge denotes a certain

pressure; then, from steam tables we can find the temperature corresponding to this pressure. But the temperature recorded by the first thermometer, will be found to be some degrees lower, the reason being radiation from the stem of the external part of the thermometer. This radiation we can also assume to exist to the same extent between the two thermometers, and by adding the deficiency of the first thermometer compared to the temperature corresponding to the reading of the first pressure gauge, we obtain the temperature of the steam after passing through the orifice with adiabatic expansion. Now, if the temperature denoted by the second thermometer is higher than that corresponding to the pressure denoted by the second pressure gauge, the steam is superheated and the difference between the two temperatures is the amount of superheat.

From the relation:—

$$\int_{32}^{T_1} \frac{c\,dt}{T} + \frac{x_1 r_1}{T_1} = \int_{32}^{T_2} \frac{c\,dt}{T} + \frac{x_2 r_2}{T_2}$$

where the integral denotes the entropy of the liquid; x_1, the portion of steam in a mixture of one pound of steam and water before passing the orifice; r_1, the corresponding heat of evaporization of the steam; T_1 and T_2, respective absolute temperatures; the second integral and the other term being the similar terms for the other side of the orifice. In case there is so much water present in the steam that the latter is not superheated by throttling when passing through the orifice, x_2, the quality of the steam, can be found from the above relation; x_1 being measured as previously described by determining the moisture or quality of the steam before it reaches the orifice.

There might seem a fault in the apparatus at first sight; e.g., when starting up a test, water will collect near the plate containing the orifice. But this can not be avoided no matter what position the plate will occupy; for if the rest of the piping beginning from the first oil-well be made vertical, the same trouble will still exist, and to a greater extent, for

low pressure steam will have to drive or lift the accumulated water up into the condenser. The way this trouble can be eliminated is, not to start the test for about an hour after steam was admitted into the apparatus, the lower valve, or the one to the right being left entirely open and no cooling or condensing water admitted into the condenser. In about a half or three quarters of an hour, steam will begin to issue from the condenser, instead of water; and after this steaming continues for about ten minutes, it is safe to start the test.

The manner of carrying on a test will best be seen from the tabulated readings.

P_1 is the reading of the first or higher gauge.

P_2 " " " " " second " lower "

T_1 and T_2, corresponding thermometer readings.

t is time.

W full is the weight of bucket with the water of condensed steam.

W emp is the weight of empty bucket.

p is the reading of the pressure gauge on the calorimeter.

t_x is the time at mark of the water in the gauge glass of the calorimeter.

The orifice in the plate is circular with square edges, and is $\frac{3}{32}''$ in diameter, so that its area is $\frac{\pi}{4} \cdot \left(\frac{3}{32}\right)^2 = .006903$ sq in.

The tests of which the tables are the readings, were divided up into half hour runs. During the first run, the lower pressure gauge was made to register zero, by opening the outer or lower valve fully. The mean of the readings, which were taken every five minutes, of the other gauge is 94.86. The barometric pressure at that time was 14.798. The mean gauge pressure was corrected from the calibration curve of the gauge, and the true pressure was found to be 94.70.

To the corrected gauge pressure, if we add the atmospheric pressure, we obtain the absolute pressure 109.5.

Assuming Napier's formula

$$W = \frac{PA}{K}$$

$$K = \frac{PA}{W}$$

In the first run, the condensed water was weighed at the end of each fifteen minutes, but it was thought that although there would be an error committed in weighing, yet, in order to elliminate an accumulated error due to water collecting at any instant in the pipes leading to the condenser, the weighing of the condensed water was divided up into three portions.

The mean discharge in fifteen minutes during the first run $= 10^{\#} - 2\tfrac{15}{16}\,oz = 10.184^{\#}$

Mean discharge in one second $= \frac{10.184}{15 \times 60}$.

$$\therefore\ K = \frac{109.5 \times .006903 \times 15 \times 60}{10.184} = 68.0$$

This value for K seems to be entirely too small compared to the following values. But the reason for this may be ascribed to not taking the precaution mentioned before; viz., not to start the test until steam was coming out of the condenser for about ten or

or fifteen minutes, before turning on the water for condensing it.

N. B. The Calorimeter being attached near the seperator, the quality of the steam which can be determined with it is the dryest that can be used with this apparatus; and the tests that were made were upon steam whose quality was thus determined, and <u>no</u> <u>water</u> was admitted into the steam from the reservoir.

Test 1 and 2

$P_2 = 0$ Quality = 94.9%

P_1	T_1	T_2	t	W full	W Empty	p	t_x
93	324	214	12.45		2*-0 oz	92	
93	323	221	12.50			92	$t_9 = 12.49$
94.5	324	222	12.55			93	
95.5	325	238	1.00	12*-4$\frac{5}{8}$ oz	2-$\frac{5}{8}$	95	$t_{15} = 1.00$
96	326	240	1.05			—	
96	326	242	1.10			—	
			1.15	12-2$\frac{1}{16}$		—	

$P_2 = 10$ Quality = 94.0%

100	324	250	1.20		2-$\frac{3}{4}$	95	
102	324	252	1.25			94	$t_7 = 1.23$
105	327	250	1.30			100	
105	330	250	1.35	12-$\frac{1}{2}$	2-0	105	$t_{14} = 1.37$
104	332	252	1.40			140	
102	331	255	1.45			—	
			1.50	12-6$\frac{1}{2}$		—	

Test 3 and 4

$P_2 = 20$ Quality = 93.3%

P_1	T_1	T_2	t	W full	W Emp	p	t_x
94	334	249	2.00		2 - $\frac{1}{8}$	—	
93.5	336	250	2.05			103	$t_6 = 2.05$
98	337	252	2.10	8 - 14$\frac{7}{8}$	2 - $\frac{5}{8}$	105	
103	338	251	2.15			105	$t_{14} = 2.17$
106	336	256	2.20	9 - 2	2 - $\frac{3}{16}$	103	
105	337	257	2.25			—	
			2.30	8 - 14$\frac{3}{4}$		—	

$P_2 = 30$ Quality = 94.7%

P_1	T_1	T_2	t	W full	W Emp	p	t_x
99	334	264	2.35		2 - $\frac{5}{16}$	100	
99	334	265	2.40			101	$t_6 = 2.39$
99	334	265	2.45	8 - 11$\frac{1}{2}$	2 - $\frac{3}{4}$	101	
98	333	265	2.50			100	
96	326	270	2.55	8 - 10$\frac{3}{8}$	2 - $\frac{1}{4}$	100	
96	326	270	3.00			100	$t_{16} = 2.57$
			3.05	8 - 9$\frac{3}{8}$		—	

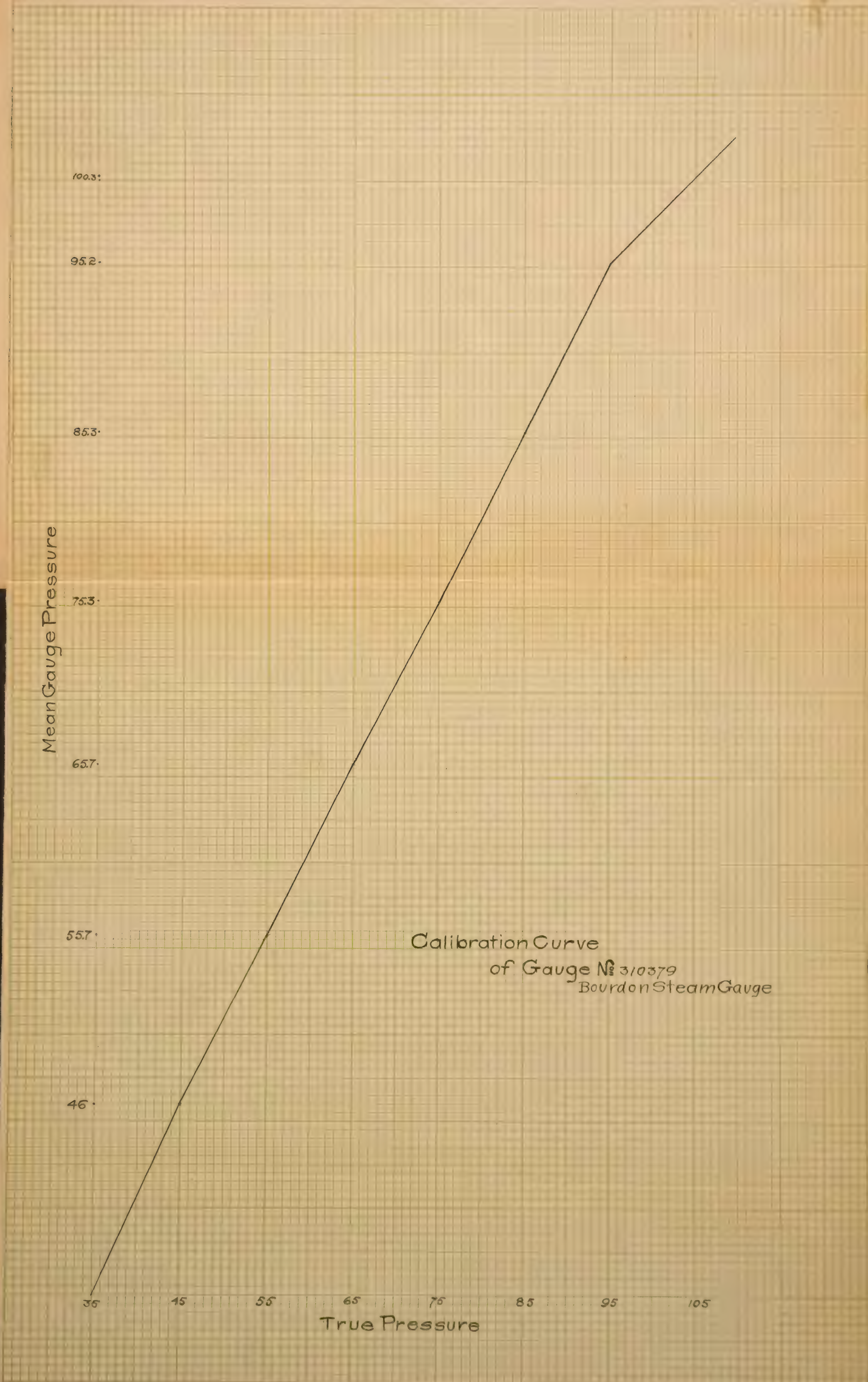
Test 5 and 6.

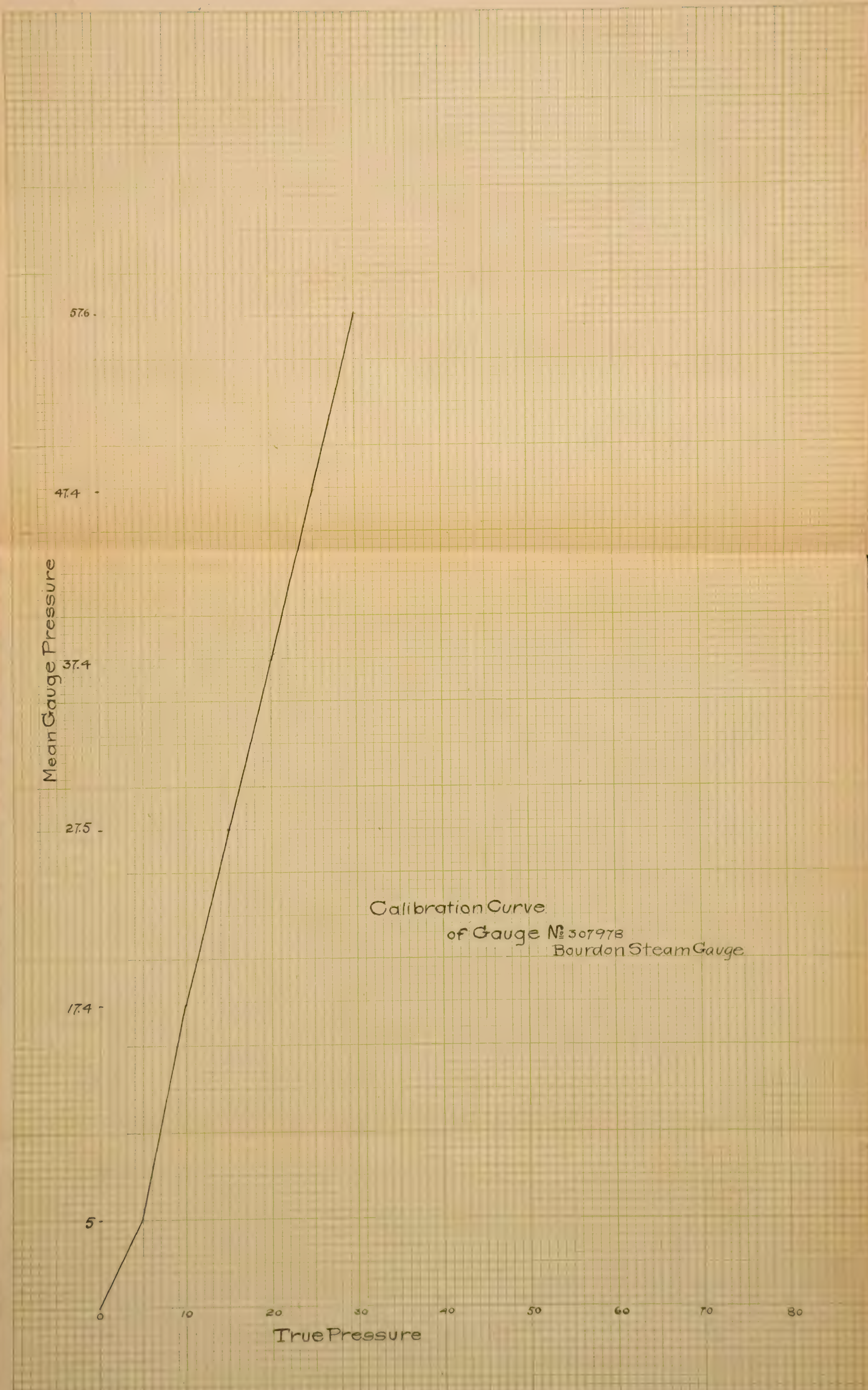
$P_2 = 35$ Quality = 93.8%

P_1	T_1	T_2	t	W full	W Emp	p	t_x
103	330	278	3.10		2-$\frac{7}{16}$	105	
100	329	277	3.15			102	t_5=3.12
100	328	276	3.20	9-0	2-1	102	
100	328	277	3.25			101	
100	327	277	3.30	8-14$\frac{3}{16}$	2-$\frac{5}{8}$	101	t_{15}=3.27
98.5	326	276	3.35			—	
			3.40	8-10$\frac{3}{8}$		—	

$P_2 = 40$ Quality = 93.8%

95	325	284	3.45		2-$\frac{3}{4}$	99	
95	325	281	3.50			99	t_4=3.48
98	326	282	3.55	8-8$\frac{1}{4}$	2-$\frac{7}{8}$	100	
99	328	283	4.00			100	
102	329	284	4.05	8-12$\frac{3}{4}$	2-$\frac{3}{4}$	103	
103	330	282	4.10			105	t_{18}=4.07
			4.15	9-0		—	


100.3
95.2
85.3
75.3
65.7
55.7
46
35
45
55
65
75
85
95
105
Mean Gauge Pressure
True Pressure
Calibration Curve
of Gauge № 310379
Bourdon Steam Gauge



Calibration Curve
of Gauge № 307978
Bourdon Steam Gauge
Mean Gauge Pressure
57.6
47.4
37.4
27.5
17.4
5
True Pressure
0
10
20
30
40
50
60
70
80